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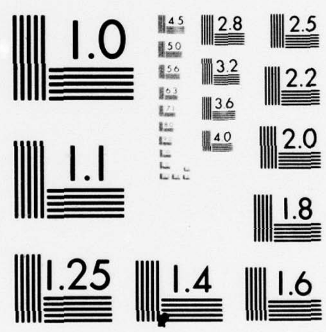

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Microstability of a Focussed Ion Beam Propagating Through a Z-Pinch Plasma

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A beam-plasma system consisting of a focussed light ion beam propagating through a z-pinch plasma is analyzed for microinstabilities. Two instabilities are discussed, one driven by the relative streaming between beam ions and electrons and the other driven by streaming between plasma ions and electrons. Conditions for stability of both modes are derived and are used to demonstrate that ion beams appropriate for use in a pellet fusion device can be propagated to the pellet through a z-pinch plasma without disruptive microturbulence.

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CONTENTS

MICROSTABILITY OF A FOCUSSED ION BEAM PROPAGATING THROUGH A Z–PINCH PLASMA

I. INTRODUCTION

In order to utilize intense ion beams in a pellet fusion device, it is generally considered necessary to propagate the beam a distance L (about 10 m) from the acceleration region while employing some focussing scheme to deliver the beam to the target. For heavy ion beams (e.g. 40 GeV U^+ ions), field-free ballistic focussing[1,2] to the target has been proposed. For light ion beams (e.g. 10 MeV protons), one concept[3] involves focussing the beam just outside the diode region by known techniques[4] and then injecting the focussed beam into a z-pinch plasma channel. The beam remains focussed during propagation by the magnetic field in the plasma channel and is delivered on target with no additional focussing. This scheme for light ion beams is considered in this paper. Since the plasma channel must remain intact during transit of the beam, a knowledge of the dependence of the linear growth rates of unstable perturbations on the parameters of the system is essential. Here the concern will be with microinstabilities.

It will be assumed that the z-pinch channel is produced in a MHD stable configuration. The question of the effects that the passage of the beam may have on the MHD equilibrium and stability of the beam-

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plasma system is an important one. For example, the primary and return current heating of the plasma by the beam may significantly modify the pressure profile of the background plasma and thus upset the MHD equilibrium. This question is not addressed here, but instead it will be assumed that the plasma channel remains in a stable MHD equilibrium during the passage of the beam in order to study the microinstabilities of the system.

Because the ion beam streams through the background plasma, the system may be subject to velocity-space instabilities. Two modes will be investigated. One mode, which will be referred to as the e-b mode, involves the interaction of beam ions and background electrons and the other mode, the e-i mode, involves the interaction of background electrons and background ions. Relative motion between the stationary background ions and the background electrons arises from both the z-pinch electron current and the electron return current driven by the ion beam; however, the return current provides the larger contribution since the ion-beam current greatly exceeds that establishing the channel.

An electrostatic stability analysis is used for simplicity and the wavevector $\underset{\sim}{k}$ is chosen to be parallel to the direction of beam propagation (i.e. $\underset{\sim}{k} = k\hat{e}_z$). If either of the two modes that are considered is sufficiently unstable to create strong microturbulence, the resulting anamolous resistivity could seriously affect the current neutralization of the beam which, in turn, might disrupt beam transport because of the

perturbed magnetic guide field. Thus it is important to know in which parameter regimes these modes are stable in order to design a system which will maintain good current neutralization. It should be noted that the high density of the z-pinch plasma required to insure good beam neutralization[5] ($n \simeq 5 \times 10^{18}$ cm^{-3}) implies that collisional processes will play an important role in the stability analyses.

Two models will be considered for the z-pinch plasma channel. In Section II, a surface current model will be used to describe the plasma channel and in Section III a uniform current model will be used. Although a uniform current model more closely approximates the z-pinch plasma which is envisioned in this focussed ion beam propagation scheme, the surface current model is treated here as well for the sake of clarity. In both cases, expressions for the linear growth rates of the e-b mode and the e-i mode will be derived. From these results, it will be demonstrated that it is possible to propagate a focussed ion beam without the development of microinstabilities. This conclusion and other results will be discussed in Section IV.

II. MICROSTABILITY ANALYSIS - SURFACE CURRENT MODEL

The case where the z-pinch current flows on the surface of the plasma column will be considered first. For simplicity, no applied axial magnetic field will be considered. Thus the beam propagates in a field-free region and beam ions only encounter the azimuthal field, B_θ, when they are at the edge of the beam. Writing the field as

$$B_\theta = \begin{cases} 0 & r < a \\ B_p a/r & r > a \end{cases} \quad , \tag{1}$$

where B_p is the magnitude of the peak magnetic field, the vector potential, $\underset{\sim}{A}$, becomes

$$\underset{\sim}{A} = \hat{\underset{\sim}{e}}_z \left\{ \begin{array}{ll} 0 & r < a \\ -B_p a \ln (r/a) & r > a \end{array} \right\} \quad . \tag{2}$$

The beam ions have straight line orbits in the interior of the beam, and are reflected back into the interior when they encounter the azimuthal field at the edge of the beam. If the beam distribution function is written as

$$f_b = N\exp\left[-\left(\frac{m_b v^2}{2} - V_z P_z\right) \Big/ T_b\right] \quad , \tag{3}$$

where V_z is a constant streaming velocity of the beam, $P_z = m_b(v_z + eA_z/m_b c)$ is the axial canonical momentum and T_b is the beam thermal energy, then from Eq. (2)

$$f_b = n_b\left(\frac{m_b}{2\pi T_b}\right)^{3/2} \exp\left\{-\frac{m_b}{2T_b}\left[v_r^2 + v_\theta^2 + (v_z - V_z)^2\right]\right\} \left\{ \begin{array}{ll} 1 & r < a \\ (\frac{a}{r})^\beta & r > a \end{array} \right\} , \tag{4}$$

where n_b is the beam density in the interior of the beam, $\beta \equiv 2V_z\omega_{cb}a/u_b^2$, $\omega_{cb} \equiv eB_p/m_b c$ and $T_b \equiv m_b u_b^2/2$. The condition for the beam to be well confined by B_θ (i.e. the beam density falls off rapidly for $r > a$) is just

$$\beta > 1 \quad . \tag{5}$$

The same condition on β is obtained from the beam-ion orbit equations by requiring that $(r_m - a)/a < 1$, where r_m is the maximum radial excursion for any given beam ion.

For a 10 MeV proton beam ($V_z = 4.4 \times 10^9$ cm/sec) with a thermal energy of $T_b = 0.1$ MeV ($u_b = 4.4 \times 10^8$ cm/sec), Eq. (5) shows that $B_p > 4.2$ kG is required in order to propagate a focussed beam of radius $a = 0.5$ cm. Typically, $B_p \simeq 40$ kG will be used (i.e. $\beta \simeq 10$). A typical system will also have a beam density of $n_b \leqslant 1 \times 10^{16}$ cm^{-3} (i.e. $n_b/n_p \ll 1$). Thus, with full current neutralization the electrons drift relative to the stationary ions with an axial velocity of $V_e \simeq n_b V_z/n_p \simeq 9 \times 10^6$ cm/sec.

The conservation of axial canonical momentum shows that a beam ion moves with a larger axial speed when $r > a$ than when $r < a$. In fact, for $r > a$

$$\Delta v_z \equiv v_z - V_z = \omega_{cb} a \ell n (r/a). \qquad (6)$$

During the time it takes for a beam ion to reflect off the magnetic field barrier ($r > a$), this increased axial speed moves the beam ion a distance $\Delta Z \simeq 4 a u_b \beta / V_z$ ahead of the position it would have if it were streaming in the interior of the beam ($r < a$) with a speed V_z. If $k\Delta z > 1$, then this process is equivalent to an effective collisional process knocking a beam ion out of phase with a wave moving with a phase velocity $\omega/k \simeq V_z$ each time the beam ion reflects off the magnetic field. This process is not important here because the effective collision time, $\tau \simeq a/u_b$, is much longer than the time scales of

interest for the e-b mode, however, a similar process will be important when considering the e-b mode for the uniform current case discussed in Section III, since in that case the beam ion motion is influenced by B_θ at all r, not just r > a.

Considering the above results, a beam-plasma system with the Z-pinch current flowing on the surface of the plasma channel can be assumed to be infinite and uniform for the purposes of this stability analysis. This is also a reasonable assumption since it can be shown that ka >> 1 in the parameter regimes of interest. The electrons are treated as a cold collisional fluid and the beam ions are treated kinetically. The appropriate dispersion equation for the e-b mode is (see Appendix A)

$$1 = \frac{\omega_{pe}^2}{\omega(\omega+i\nu_e)} - \frac{2\omega_{pb}^2}{k^2u_b^2}\left\{1 + \frac{i\sqrt{\pi}(\omega-kV_z)}{ku_b}\exp\left[-\left(\frac{\omega-kV_z}{ku_b}\right)^2\right]\right\}, \qquad (7)$$

where $\nu_e = 1.5 \times 10^{-6}\, n_e\lambda_{ei}/T_e^{3/2}$ is the electron collision frequency, λ_{ei} is the Coulomb logrithm for electron-ion collisions and $\omega_{p\alpha}$ is the plasma frequency for species α (T_e is expressed in eV). Solving for the maximum growth rate ω_i^m, of the e-b mode, one obtains

$$\omega_r = \omega_{pe}\left(1 - \frac{\omega_{pb}^2}{k^2u_b^2}\right) \qquad (8)$$

$$k^m = \frac{\omega_{pe}}{V_z}\left(1 + \frac{u_b}{\sqrt{2}V_z}\right) \qquad (9)$$

and
$$\omega_i^m = -\frac{\nu_e}{2} + \sqrt{\frac{\pi}{2}}\,\frac{\omega_{pb}^2}{\omega_{pe}}\,\frac{V_z^2}{u_b^2}\exp(-1/2) \tag{10}$$

where $\omega = \omega_r + i\omega_i$. The condition for stability of the e-b mode is just

$$\frac{\exp(1/2)}{\sqrt{2\pi}}\,\frac{\nu_e\omega_{pe}}{\omega_{pb}^2}\left(\frac{u_b}{V_z}\right)^2 > 1 \quad . \tag{11}$$

For the parameters used previously, the e-b mode is stable for $T_e \leqslant 50$ eV. Since it is expected[6] that the primary and return current heating will raise the electron temperature no higher than this value, the e-b mode is stable for the case of surface currents.

Because good current neutralization requires $n_b/n_p \ll 1$, the streaming velocity, V_e, of the electrons relative to the stationary background ions is actually subthermal ($V_e < u_e$, where $T_e = m_e u_e^2/2$) for $T_e \geqslant 1$ eV. Thus, the electrons can be considered as a warm collisional fluid when studying the e-i mode. Since $T_e \geqslant T_i (u_i \ll u_e)$, the ions can be treated as a warm, collisional fluid as well. In the momentum transport equations, the collisional drag force on species α is expressed as $-m_\alpha n_\alpha \nu_{\alpha\beta}(\underset{\sim}{v}_\alpha - \underset{\sim}{v}_\beta)$ and the thermal pressure gradient term is expressed as $-T_\alpha \underset{\sim}{\nabla} n_\alpha$. Thus the dispersion equation for the e-i mode becomes

$$1 = \frac{\omega_{pi}^2(\omega_d^2 - k^2u_e^2/2) + \omega_{pe}^2(\omega^2 - k^2u_i^2/2)}{\omega\omega_d[\nu_e\nu_i + (\omega + i\nu_i - k^2u_i^2/2\omega)(\omega_d + i\nu_e - k^2u_e^2/2\omega_d)]} \tag{12}$$

where $\omega_d \equiv \omega - kV_e$, $\nu_i = m_e\nu_e/m_i$ and ν_e was defined earlier (see Appendix A for details of the derivation).

Solving Eq. (12) for the real and imaginary parts of ω, one obtains for $T_i < T_e$

$$\omega_r = kC_s/(1 + k^2\lambda_d^2)^{\frac{1}{2}} \quad , \tag{13}$$

and

$$\omega_i = \frac{-\nu_i k^2\lambda_d^2}{2(1 + k^2\lambda_d^2)}\left[1 + \frac{V_e/C_s - 1/(1 + k^2\lambda_d^2)^{\frac{1}{2}}}{(1 + k^2\lambda_d^2)^{\frac{1}{2}}}\right] \quad , \tag{14}$$

where $C_s^2 \equiv T_e/m_i$ is the ion sound speed and $\lambda_d^2 = u_e^2/2\omega_{pe}^2$ is the electron Debye length. In deriving this result it was assumed that $k^2u_e^2 > \nu_e\omega_d > \omega_d^2$ and $\omega > ku_i > \nu_i$. Since

$$\omega_i < 0 \qquad \text{for all } k \tag{15}$$

the e-i mode is stable.

Summarizing the results obtained in this section, it was found that e-i mode is stabilized by collisional damping and that the e-b mode is stable because the electrons are not heated above a critical temperature determined by Eq. (11) for the parameters of interest. These results apply to the case where the z-pinch current flows on the surface of the plasma column. If the z-pinch current is distributed uniformly across the plasma channel the azimuthal magnetic field within the channel can have an additional stabilizing influence.

III. MICROSTABILITY ANALYSIS - UNIFORM CURRENT CASE

If the z-pinch current is uniformly distributed, then the azimuthal magnetic field increases linearly with radius inside the beam. The beam ion orbits are now nearly sinusoidal when the ion enters the channel at a small angle to the plasma axis. For mathematical convenience, a slab model will be used here. This is appropriate for beam ions with small angular momentum since motion then occurs in a plane. If the beam is streaming in the z direction, then the aximuthal field will point in the y direction and can be written as

$$B_y = \begin{cases} B_p x/a & -a < x < a \\ B_p & |x| > a \end{cases} \tag{16}$$

The vector potential, $\underset{\sim}{A}$, in this geometry becomes

$$\underset{\sim}{A} = \hat{\underset{\sim}{e}}_z \begin{cases} -B_p x^2/2a & -a < x < a \\ -B_p(x-a/2) & |x| > a \end{cases} . \tag{17}$$

Again writing the distribution function as in Eq. (3), f_b can be put in the following form

$$f_b = n_b\left(\frac{m_b}{2\pi T_b}\right)^{3/2} \exp\left\{-\frac{m_b}{2T_b}[v_x^2+v_y^2+(v_z-V_z)^2]\right\} \begin{cases} \exp(-\frac{\beta x^2}{2a^2}) & -a < x < a \\ \exp[-\frac{\beta}{a}(x-\frac{a}{2})] & |x| > a \end{cases} , \tag{18}$$

where again $\beta \equiv 2V_z\omega_{cb}a/u_b^2$, $\omega_{cb} = eB_p/m_bC$ and again the condition for confinement of the beam is just $\beta/2 \geq 1$. The beam ion orbits are affected by the axial electric field ($|E_z| = m_e\nu_e V/e$) which is required to drive the z-pinch current, however, on time scales shorter than the beam pulse length ($\tau < 50$ nsec) the effect of E_z is negligible. Using Eq. (16) for B_y, the beam ion equation of motion can be solved approximately in terms of trigonometric functions.

$$x' = x\cos\omega_o\tau + \frac{v_x}{\omega_o}\sin\omega_o\tau \quad , \tag{19}$$

$$y' = y + v_y\tau \quad , \tag{20}$$

$$z' = z + \left[v_z + \frac{\omega_o^2}{4V_z}\left(\frac{v_x^2}{\omega_o^2} - x^2\right)\right]\tau + \frac{\omega_o}{8V_z}\left(x^2 - \frac{v_x^2}{\omega_o^2}\right)\sin 2\omega_o\tau - \frac{xv_x}{4V_z}(\cos 2\omega_o\tau - 1), \tag{21}$$

and

$$v_x' = v_x\cos\omega_o\tau - x\omega_o\sin\omega_o\tau \quad , \tag{22}$$

$$v_y' = v_y \quad , \tag{23}$$

$$v_z' = v_z - \frac{\omega_o^2}{4V_z}\left(\frac{v_x^2}{\omega_o^2} - x^2\right)(\cos 2\omega_o\tau - 1) + \frac{\omega_o x v_x}{2V_z}\sin 2\omega_o\tau \quad , \tag{24}$$

where $\omega_o \equiv (\omega_{cb}V_z/a)^{\frac{1}{2}}$ and $(\underset{\sim}{x}', \underset{\sim}{v}') = (\underset{\sim}{x}, \underset{\sim}{v})$ at $\tau = t' - t = 0$. This solution is obtained by replacing v_z' with v_z in the x-component of the equations of motion and is therefore applicable when $v_x/v_z \ll 1$. In this case, an expression for Δv_z (averaged over a $2\omega_o$ period) can be obtained from Eq. (24),

$$\Delta v_z \equiv \overline{|v_z' - v_z|} \simeq \frac{\omega_{cb}a}{4} \quad , \tag{25}$$

where, for example, an ion starting at $\tau = 0$ from the point of maximum excursion ($x' = x = a$ and $v_x' = v_x = 0$) will have an axial velocity given by $v_z'(\tau) = v_z + (a^2\omega_o^2/4V_z)(\cos 2\omega_o\tau - 1)$. Using the parameters from the example in Section II, as well as Eq. (9) for k^m derived for the e-b mode, it is found that $k^m\Delta v_z \gg \omega_i^m$, so that the average beam ion moves out of phase with the wave even before one e-folding time. In addition, $\omega_o L/V_z \gg 1$, so that the beam ions complete many oscillations while propagating down the plasma channel. Because of the oscillatory motion in the z-direction the beam ions are constantly moving in and out of phase with waves moving with phase velocity $\omega_r/k \simeq V_z$. This process is responsible for a reduction in the growth rate of the e-b mode, which will now be derived.

Using the method of characteristics[7], the perturbation to the beam distribution function can be written as

$$f_{b1}(\underset{\sim}{x},\underset{\sim}{v},t) = \frac{+q_b}{m_b} \int_{-\infty}^{0} \left(\frac{\partial\bar{\phi}}{\partial x'} \frac{\partial f_{bo}}{\partial v_x'} + ik_z\bar{\phi} \frac{\partial f_{bo}}{\partial v_z'} \right) \exp[i(kZ - \omega\tau)]d\tau \quad , \tag{26}$$

for electrostatic perturbations of the form $\phi = \bar{\phi}(x)\exp[i(kz-\omega t)]$. Here $Z = z' - z$, $k_y = 0$ and the x dependence has not been Fourier analyzed. Since the solution of the eigenfunction equation for $\bar{\phi}(x)$ is difficult and of little present value, the solution will be approximated by

$$\bar{\phi}(x) = \bar{\phi} \begin{Bmatrix} 1 - \dfrac{x^2}{a_b^2} & -a_b < x < a_b \\ \\ 0 & |x| > a_b \end{Bmatrix} , \qquad (27)$$

so that the perturbation field is confined within the beam. Here $a_b \equiv (2/\beta)^{\frac{1}{2}}a$ is the beam radius and $a_b < a$ for $\beta > 2$. The perturbation is chosen to peak on axis in order to model the zero-order eigenmode (no nodes). The expression for f_{b1}, Eq. (26), is now integrated over velocity space to obtain an expression for the beam density perturbation, n_{b1}, which is inserted into Poissons equation

$$-\nabla^2\phi = 4\pi e(n_{b1} - n_{e1}) \qquad , \qquad (28)$$

along with the appropriate expression for n_{e1}. Again the electrons are taken to be a cold collisional fluid when treating the e-b mode.

Taking a weighted spatial average of Poisson's equation results in an approximate algebraic dispersion equation

$$1 = \frac{\omega_{pe}^2}{\omega(\omega+i\nu_e)} - \frac{2\omega_{pb}^2 R_b}{k^2u_b^2}\left\{1 + \frac{i\sqrt{\pi}(\omega-kV_z)}{ku_b}\exp\left[-\left(\frac{\omega-kV_z}{ku_b}\right)^2\right]\right\} . \qquad (29)$$

The details of the calculations leading to this dispersion equation are given in Appendix B.

Eq. (29) has the same form as Eq. (7) except that ω_{pb}^2 is replaced $\omega_{pb}^2 R_b$ where R_b reflects the reduction of beam ions in resonance with the wave as discussed earlier.

$$R_b = 8\alpha_o \omega_o V_z / k u_b^2 \quad . \tag{30}$$

Note that R_b is the ratio of the wavelength of the perturbation to the amplitude of the oscillatory motion in the z direction of the beam ions (see Eq. (21) for the oscillations in z at $2\omega_o$). Also note that for the collisional electrons $|\omega(\omega+i\nu_e)| >> |eB_p/m_e c|^2$.

The condition for stability of the e-b mode for the uniform current case becomes

$$\frac{\exp(\frac{1}{2})}{8\sqrt{2\pi}\ \alpha_o} \left(\frac{\nu_e \omega_{pe}^2}{\omega_o \omega_{pb}^2}\right)\left(\frac{u_b}{V_z}\right)^4 > 1 \quad . \tag{31}$$

In this case the e-b mode is stable for $T_e \leqslant 1.0$ keV when the parameters used in the example in Section II are inserted into Eq. (31). This is well above the temperatures that are expected. Thus the uniform current case is more desirable than the surface current case with respect to the stability of the e-b mode.

The analysis of the e-i mode for the uniform current case differs very little from the analysis for the surface current case since $\omega_{ce} << ku_e$, ω, ν_e and $\omega_{ci} << \omega$, ν_i. Although the collisional fluid treatment of the electrons and ions do not include finite Larmor

radius effects, such effects will only further stabilize the system. Thus the e-i mode will be stable for the uniform current case just as in the surface current case.

IV. CONCLUSIONS

The microstability analysis presented here treated the e-b and e-i modes of a focussed ion beam-plasma system. The high density ($n_p \sim 5 \times 10^{18}$ cm^{-3}) required for the background z-pinch plasma provides for a high frequency of collisions between the background electrons and ions. It was found that this results in the stabilization of the e-b mode by collisional damping, even for reasonably high electron temperatures. In the example presented in Section II, the e-b mode was stable for $T_e \leq 50$ eV for the surface current case. When the z-pinch current is distributed uniformly over the plasma channel, it was found that the e-b mode was even further stabilized by the oscillatory motion (in the direction of beam propagation) exhibited by the beam ions. In this case the e-b mode was stable for $T_e \leq 1.0$ keV for the same parameter as used in the previous example. Since the electrons are expected to be heated no higher than 50 eV, z-pinch current flow in the plasma channel ensures stability of the e-b mode. Uniform current flow is anticipated, because the magnetic diffusion time in a current channel 1 cm in diameter is much less than the many μsec required to establish the channel. The low channel currents required to establish the 40 kG azimuthal fields are sufficient to heat the channel to more than a few eV before the ion beam enters the channel.

Electron-ion collisions are also responsible for stabilization of the e-i mode. The condition $n_b/n_p \ll 1$, required for good charge and current neutralization of the beam, implies the $V_e < u_e$ for a typical system. This is important since the e-i mode can be unstable if $V_e > u_e$. The results for the e-i mode are basically unchanged if the current flows in the interior of the beam channel.

Thus both the e-b and e-i mode are stable for a typical system if the focussed ion beam is propagated through a z-pinch plasma channel where the z-pinch current flows in the interior of the channel.

APPENDIX A

Treating the plasma electrons and ions as collisional fluids, the linearized fluid equations describing their motion under the influence of an electrostatic perturbation of the form exp $i(kz-\omega t)$ become

$$(\omega - kV_\alpha)n_{\alpha 1} = n_\alpha \underset{\sim}{k}\cdot\underset{\sim}{v}_{\alpha 1} \tag{A.1}$$

and

$$m_\alpha(\omega - kV_\alpha)\underset{\sim}{v}_{\alpha 1} = q_\alpha \underset{\sim}{k} n\phi_1 + T_\alpha \underset{\sim}{k} n_{\alpha 1}/n_\alpha - i\nu_\alpha m_\alpha(\underset{\sim}{v}_\alpha - \underset{\sim}{v}_\beta)_1 , \tag{A.2}$$

where V_α is the streaming velocity, T_α is the thermal energy and ν_α is the collision frequency for species $\alpha(\alpha = e,i)$, and the subscript "1" signifies a perturbation quantity. Solving for $n_{\alpha 1}$ in terms of ϕ_1

$$n_{\alpha 1} = \frac{n_\alpha q_\alpha k^2\phi_1/m_\alpha(\omega - kV_\alpha)}{\left(\nu_e\nu_i + \left[\omega + i\nu_i - k^2u_i^2/2\omega\right]\left[\omega - kV_e + i\nu_e - k^2u_e^2/2(\omega - kV_e)\right]\right)} , \tag{A.3}$$

where $V_i = 0$ and $T_\alpha \equiv m_\alpha u_\alpha^2/2$. This expression for $n_{\alpha 1}$, Eq. (A.3), is inserted into Poisson's equation in order to obtain the dispersion equation.

For the e-b mode the beam ions are treated kinetically so that

$$n_{b1} = \frac{-q_b}{m_b}\int \frac{\underset{\sim}{k}\cdot\partial f_b/\partial \underset{\sim}{v}}{\omega - \underset{\sim}{k}\cdot\underset{\sim}{v}}\,\phi_1 d^3\underset{\sim}{v} , \tag{A.4}$$

where f_b is given in Eq. (4). Rewriting Eq. (A.4) in terms of the plasma dispersion function, $Z(\xi)$, results in

$$n_{b1} = \frac{-2q_b n_b}{m_b u_b^2} [1 + \xi Z(\xi)] \quad , \tag{A.5}$$

where $\xi \equiv (\omega - kV_z)/ku_b$. For a warm beam $\xi < 1$ and the power series expansion of $Z(\xi)$ is appropriate[8].

APPENDIX B

The ion beam contribution to Poisson's equation, Eq. (28), is given by

$$n_{b1} = \int d^3v\, f_{b1} \quad , \tag{B.1}$$

where f_{b1} is given in Eq. (26). Taking the weighted spatial average of Poisson's equation, the desired expression for the dispersion equation, Eq. (29), becomes

$$\langle n_{b1} \rangle \equiv \int_{-a_b}^{a_b} n_{b1}\left(1 - \frac{x^2}{a_b^2}\right)\frac{dx}{a_b} \quad . \tag{B.2}$$

Assuming $\beta/2 > 1$, so that for all x

$$f_b \simeq n_b\left(\frac{m_b}{2\pi T_b}\right)^{3/2} \exp\left(-\frac{\beta x^2}{2a^2}\right) \exp\left\{-\frac{m_b}{2T_b}\left[v_x^2 + v_y^2 + (v_z - V_z)^2\right]\right\} \quad , \tag{B.3}$$

Eq. (B.2) becomes

$$\langle n_{b1} \rangle = \frac{en_b}{T_b a_b}\left(\frac{m_b}{2\pi T_b}\right)^{3/2} \int_{-a_b}^{a_b} dx\left(1 - \frac{x^2}{a_b^2}\right) \exp\left(-\frac{\beta x^2}{2a^2}\right)\int d^3v \int_{-\infty}^{0} d\tau\left[\left(-\frac{2x'}{a^2}\right)v_x'\right.$$

$$\left. + ik\left(1 - \frac{x'^2}{a^2}\right)\left(v_z' - V_z\right)\right] \exp\left\{-\frac{m_b}{2T_b}\left[v_x^2 + v_y^2 + (v_z - V_z)^2\right]\right\} \exp\left[i(kZ - \omega\tau)\right] \quad , \tag{B.4}$$

where $Z = z' - z$, $\tau = t' - t$, and $\underset{\sim}{x}'$ and $\underset{\sim}{v}'$ are given in Eqs. (19)-(24). Then using the Bessel function indentities[9]

$$e^{\pm ia \sin bt} = J_o(a) + 2\sum_{m=1}^{\infty} J_{2m}(a)\cos(2mbt) \pm 2i\sum_{m=1}^{\infty} J_{2m+1}(a)\sin[(2m+1)bt] \tag{B.5}$$

and

$$e^{ia \cos bt} = J_o(a) + 2\sum_{m=1}^{\infty} i\, J(a)\cos(mbt) \tag{B.6}$$

to simplify $\exp[ikZ]$, one obtains a sum of integrals, where each integral in the sum has the form

$$I(p,s;\eta,\mu,\sigma) \equiv \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dv_x \left[\left(\frac{x}{a}\right)^p \left(\frac{v_x}{u_b}\right)^s \exp\left(-\frac{\beta x^2}{2a^2}\right) \exp\left(-\frac{v_x^2}{u_b^2}\right)\right.$$

$$\left. \times\, J_\eta\left(\frac{v_x^2}{V_x^2}\right) J_\mu\left(\frac{2v_x u_b x}{V_x^2 a}\right) J_\sigma\left(\frac{u_b^2 x^2}{V_x^2 a^2}\right) \exp\left(\frac{iksv_x}{4V_z}\right)\right] \quad , \tag{B.7}$$

where $V_x^2 \equiv 8\omega_o V_b/k$ and where the integration over x has been extended to infinity since $\exp(-\beta x^2/2a^2) \to 0$ for $|x| > a_b \equiv (2/\beta)^{\frac{1}{2}}a$ when $\beta/2 > 1$; also p,s,η,μ and σ are all integers. Because of the oscillatory nature of the Bessel functions and the properties of the exponential function, the major contribution to I occurs at $x \approx 0$ and $v_x \approx 0$. In fact, $I(o,o;\eta,\mu,\sigma) \gg I(p,s;\eta,\mu,\sigma)$ for any $p > 0$ and $s > 0$. Furthermore, the concern is with the resonance at $\omega - kV_z \pm 2m\omega_o$ where $m = 0$. The largest contribution at this resonance is obtained when $\eta = \mu = \sigma = 0$. Performing the integration

$$I(o,o;o,o,o) = \sqrt{\pi}\, V_x^2 a_b \alpha_o / u_b \tag{B.8}$$

where

$$\alpha_o \equiv \frac{4}{\sqrt{\pi}} \int_0^\infty ds \int_0^\infty dt \left\{ \exp\left[-\frac{V_x^2}{u_b^2}\left(\frac{\beta t^2}{2} + s^2\right)\right] J_o(s^2) J_o(t^2)\, J_o(2st)\cos(2st) \right\} \tag{B.9}$$

For typical values of β and V_x/u_b (β = 10.0 and $V_x^2/u_b^2 \simeq 1.2 \times 10^{-2}$ for a_b = 0.5 cm, $B_p \simeq$ 40 kG and all other parameters the same as used previously), α_o = 0.44.

After performing the remaining τ, v_y and v_z integrations in that order, one finds

$$4\pi e \langle n_{b1} \rangle \simeq \frac{-\omega_{pb}^2}{u_b^2} \langle \phi \rangle \left[1 + \xi Z(\xi)\right] \left[\alpha_o \frac{V_x^2}{u_b^2} + O\left(\frac{V_x^4}{u_b^4}\right)\right] \tag{B.10}$$

where $\xi \equiv \omega - kV_z/ku_b$ and $Z(\xi)$ is the usual plasma dispersion function[8]. The important result here is that ion beam contribution to the dispersion equation is reduced by a factor of $V_x^2\alpha_o/u_b^2$, where $V_x^2/u_b^2 \ll 1$. Terms of $O(v_x^4/u_b^4)$ or smaller may be ignored.

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